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ABSTRACT

This final report describes a research project in sociology of education. The general objective of the project was to develop a framework for analyzing and conceptualizing multi-level social processes that will allow researchers to examine and represent school and classroom socialization structures and their effects. The framework is based on a specialized type of linear statistical model. To develop the framework, the authors built upon previous conceptualizations put forth by several investigators and have carried those ideas forward. It draws mainly on the earlier work of Duncan (1972) and of Hauser (1968), on previous work by Fennessey (1970), and on some more recent work by Cronbach and his associates (1976). Two more specific objectives of the project were (1) to examine and evaluate alternative adjustment strategies proposed for use in multi-level analysis, and (2) to explore the benefits of regarding the individual within-school regression coefficients as a vector variable characterizing the schooling process. The report contains three sections. Section 1 briefly introduces the project. Section 2 describes in some detail the background of the work in developing the general analysis framework. Section 3 gives the main elements presently included in the framework, some areas in which progress was made, and other areas in which problems were encountered. (Author/NE)

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ED220350

Multi-Level Analysis Methodology

Final Report

Grant No. NIE-G-76-0092

November 1976 - June 1978

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Section 1. Introduction

In this project, we have pursued several distinct objectives. One of these is quite general, and two others are relatively specific. The general objective has been to develop and articulate a fairly broad, yet pragmatic, framework for conceptualization and analysis of multi-level social processes. With such a framework, researchers can better represent and examine school and classroom socialization structures and their effects. Such a framework must be broad enough to allow clear, faithful representation of a wide variety of mechanisms that may be postulated as being involved in such multi-level social systems. But, it must also be pragmatic, as the title of our original proposal indicated. That is, it must use only those kinds of data that are realistically available, and only the techniques of statistical analysis which are presently feasible.

In our work to develop such a framework, we have built upon previous conceptualizations put forth by several investigators of these topics, and have carried those ideas forward somewhat. Thus, we have made considerable progress in the articulation of issues and development of a general framework. Regrettably, there are several issues on which our work has had to be left in an unfinished state. More serious, there are several areas in which we judge that serious epistemological difficulties confront anyone who seeks a comprehensive yet pragmatic multi-level analysis framework. Thus, although the present document is administratively the final report of one funded research project in sociology of education, it is more appropriately viewed intellectually as a progress report on one topic of broad social science methodological investigation.

Section 2 of this report describes in some detail the background of our work in developing the general analysis framework. Section 3 gives the main elements we are presently including in the framework, some areas where we have made progress, and others where we have encountered obstacles. The framework is based upon a specialized type of linear statistical model. It draws mainly on the earlier work of Duncan (1972) and of Hauser (1968), on previous work by Fennessey (1970) and on some more recent work by Cronbach and his associates (1976).

As mentioned, two relatively specific objectives also were pursued in the present project. The first was to examine and evaluate the alternative adjustment strategies proposed for use in multi-level analysis on the one hand by David Wiley (1976), and by Fennessey (1970). Each of these strategies seeks to deal properly with the perhaps confounded presence of what can be called "compositional effects" and "contextual effects." This objective is pursued in a report presented at the 1978 meetings of the American Educational Research Association, and subsequently published as Report #250 from the Center for Social Organization of Schools (Karweit, Fennessey, and Daiger, 1978).

The second specific objective was to explore the benefits of regarding the individual within-school regression coefficients as a vector variable (i.e., as a vector outcome) characterizing the schooling process (that is, in econometric terminology, the 'structure' in each school). Results of our work on this topic, comprising a specialized application of generalized least squares, will be presented at the 1979 Annual Meeting of the American Educational Research Association. While we have not dealt exhaustively with each of these topics, our progress on each of them has met our own expectations,

and we feel some clarification of the complex patterns of multi-level effects has been achieved.

With this brief overview of the organization of this report, we turn now to the description of the approach we have adopted in seeking to create a general multi-level analysis framework.

Section 2. Toward a General Framework for Multi-Level Analysis

During the 1950s and 1960s, very few social scientists were inclined to regard multi-level survey data as in need of a specialized mode and paradigm of quantitative analysis. The emergence of explicit thinking about a multi-level analysis paradigm can be dated from work led by Dudley Duncan, and well represented in the dissertation of Robert Hauser. Within this approach, the logic of analysis of covariance was applied to data on individuals within naturally occurring groups, such as students within schools. This conceptualization helped to provide a clear, formal definition of several separate kinds of possible multi-level effects, as well as a manageable set of procedures for carrying out actual data analysis. At the same time, it extended the logic of ANCOVA and introduced some new issues. Ironically, the authors just mentioned appear to have a strong personal conviction that actual group-level effects are very rare, and very difficult to establish. Hauser has, in fact, written several papers cautioning against the facile claim that contextual effects exist in a particular situation; these have tended to dampen enthusiasm in the discipline for multi-level studies (see Hauser, 1969, 1970, 1971, 1974; Farkas, 1974).

In his dissertation, Fennessey (1970) refined and clarified somewhat the ideas put forth by Hauser, and focused particularly on two aspects of the Duncan-Hauser approach: (1) the logic underlying the covariance adjustment when the assignment to groups is not random (cf. Lord, 1967; 1969); and (2) possible responses to the frequent finding that the slopes estimated in one group are different from those estimated in other groups. Fennessey's discussion shifted away from the terminology of analysis of covariance, and used a more general viewpoint, emphasizing a complex structural equation model, in

are
which linear equations formulated separately within-groups, and between-groups.

Matters then remained essentially static for several years; multi-level analysis ceased to be an active topic of methodological investigation. This was perhaps due in considerable measure to Hauser's strong statements of caution. In 1976, a paper by Duane Alwin (SOE) clarified some algebraic equivalences between two alternative computing procedures that had been seen by many as separate techniques. The first of these, labeled by Alwin the "contextual analysis" procedure, consists of regressing an individual level outcome score upon individual level input scores and simultaneously on group averages of these input scores. The second procedure was called by Alwin the analysis of covariance method, and corresponds to the one put forth by Hauser and other writers. Alwin's demonstration removed some of the misunderstanding, but failed to advance either technique beyond the levels achieved some years earlier. Also in 1976, however, Lee Cronbach and some associates prepared a monograph which argued strongly for the importance of adopting a multi-level viewpoint in studying school processes, and which also put forth a variant and extension of the analysis of covariance framework for multi-level analysis used by the earlier writers. Cronbach's document was important not only for the intellectual contributions it made, but also because it legitimated the importance of multi-level analyses, and showed the subtlety of many of the issues involved.

In the present project, our work has drawn upon these previous efforts, and has attempted to extend the general multi-level analysis methodology in several ways. We will use the term 'linear decomposition' to refer to the general multi-level analysis technique derived from the analysis of covariance

work and from Cronbach's work. For the remainder of Section 2, we review the main elements of the linear decomposition framework for multi-level analysis as it had developed prior to our present work. In Section 3, we go on to describe some additional refinements and distinctions we have made.

2.1 The Linear Model Framework for Multi-Level Analysis

Although the methodological debates and writings just mentioned have clarified a number of points about multi-level analysis, and indicated the need for certain cautions, the paradigm or framework is still in a rapidly changing and imperfect condition. Terminology and details of procedure are not universally shared. Thus, in the next several pages, we carry out a quick review of previous work to establish the uses we adopt.

The precursors of what we are calling the linear decomposition framework for multi-level analysis include writings on topics such as structural effects (Blau, 1960), compositional effects (Davis, Spaeth, and Huson, 1961), contextual effects (Boyle, 1966), climate effects (Coleman, 1958), and others as well. Fennessey's 1968 paper on the general linear model makes brief mention of the similarities between these older approaches and that of the linear model; and work around the same time by Duncan and by Hauser likewise contributes to the shift toward a general linear model framework. As already mentioned, Fennessey's (1970) approach to the multi-level analysis issues differs from that of Hauser by putting forth the idea that multi-level analysis involves a multiple equation linear structural model. The analyst's job is to specify and estimate the variables and parameters in those linear equations in a satisfactory way, to point out limitations on the applicability of the framework, and to extend where possible its ability to represent various possible multi-level processes and patterns.

The general linear model framework is potentially applicable whenever investigation is made of several variables, some defined at an "individual" level, and others defined at a "group" level. Since there thus are two sets of variables, each defined on different entities, a question arises as to whether one or the other set is appropriate alone, or whether instead one must use all the levels together in the analysis. The recent writings of Cronbach et al (1976), and of Burstein (1975, 1976), help greatly to clarify this issue.

To discuss these ideas, it is best to make use of an example drawn from sociology of education. Consider a research project dealing with N students, and the J schools in which they are enrolled. In this example, the individuals are the students, and the groups are the schools. Suppose the researcher is seeking to estimate the impact of a number of putative influences on one particular outcome variable, namely the academic achievement score of each student. Let us denote that outcome variable as Y_{ij} for the i^{th} student in the j^{th} school.

In addition to the outcome variable of main interest, there will be several other types of variables. We follow for the moment Fennessey's 1970 proposal for naming such variables, though with some modifications. The modifications are intended to make the terms clearer when only two levels of aggregation are involved. We define an observed individual variable as one whose value is observed and recorded for each of the N individuals. This includes both the focal outcome variable, and any other variables involved as causes or as both causes and effects. In the example described, the observed individual variables are:

achievement: measured quantitatively as the score on a standardized test. This is the focal outcome variable. It is denoted as Y. It is most convenient if the outcome variable is measured on an interval scale, but not essential.

socioeconomic level: measured quantitatively by an index. This variable is an explanatory variable, and will be denoted as S or SES.

popularity: measured quantitatively by an index. This variable is an explanatory one and will be denoted as P or POP.

motivation: measured quantitatively by an index. This variable is an explanatory one and will be denoted as M or MOT.

Another type of variable appearing in the multi-level analysis is one which is derived by use of a summary measure (for example, the mean or standard deviation) applied to all the individuals in a particular group. The resulting variable characterizes the group (here, the school), and is referred to as an aggregative group variable. In the example, the average SES of all the students in the j^{th} school is an aggregative group variable characterizing the j^{th} school.

A third type of variable appearing in the multi-level analysis is the individual minus group deviation variable. This type of variable, characterizing an individual, is defined as the difference between the observed individual variable for a particular student and his school's average score on the same variable. For example, one such variable is the deviation of student i's SES from the SES of all the students in his school (the j^{th} school). This could be expressed as $(SES_{ij} - \overline{SES}_{.j})$.

Finally, there may be other variables which characterize the groups, but which are not derived by aggregating any of the observed individual variables in the model. Such variables are called global group variables. In our example, these might include:

school location: measured as a categoric variable.

size of school: measured by enrollment.

principal leadership: measured by a rating index.

Notational complexities arise in working with a multi-level analysis because the notation must serve several unconnected purposes. First, the label should have some mnemonic value to aid in recalling the substantive meaning of the variable. Second, the notation also should indicate the way in which the variable was derived from the original data. Third, the notation must be extendable to 3, 4 or any number of levels of unit. Finally, the notation should be conveniently manipulable, and adaptable to graphic and computer representations if possible. The solution suggested by Fennessey (1970) entails the establishment of a dual labeling for each variable in the model. His aim is to have one notation which recalls the substantive meaning and relation to the data, and a second which facilitates algebraic manipulations. He also proposes some path-type diagrams, though including additional conventions, as a helpful feature in understanding the general logic of a particular multi-level analysis model. We shall follow his usage, with modifications, here, and explain it as we go along.

2.2 Decomposing the Outcome Variable and Its Causes

The multi-level analysis framework evolving from the earlier works, and advanced further in the present project, rests upon certain abstract algebraic relationships which have very convenient properties. The first such relationship is a simple decomposition of the dependent or outcome variable, routinely used in analysis of variance. It can be written as:

$$Y_{ij} \equiv \bar{Y}_{.j} + (Y_{ij} - \bar{Y}_{.j}). \qquad \text{eq. 1}$$

Equation 1 can also be written as follows:

$$YOBS(I,J) \equiv YAVG(J) + YDEV(I,J).$$

Equivalently, if one is working with deviations from the overall mean,

$$Y_{ij} \equiv (Y_{ij} - \bar{Y}_{.j}) + (\bar{Y}_{.j} - \bar{Y}_{..}) + \bar{Y}_{..}$$

This equation can also be written as follows:

$$YOBS(I,J) \equiv YGAVG + YDEV(J) + YDEV(I,J).$$

This relationship is an identity. That is, it is exact, non-causal, and tautological, and involves no assumptions whatever about probability distributions. It may be interpreted as a statement that explaining the two terms on the right hand side of equation 1 is the same as explaining the term on the left hand side of the same equation. An important feature of the particular decomposition in equation 1 (and its alternate form) is that the terms on the right hand side are orthogonal to each other. That is,

$$\sum_j \sum_i (\bar{Y}_{.j})(Y_{ij} - \bar{Y}_{.j}) = 0 = \sum_j \sum_i (\bar{Y}_{.j} - \bar{Y}_{..})(Y_{ij} - \bar{Y}_{.j}).$$

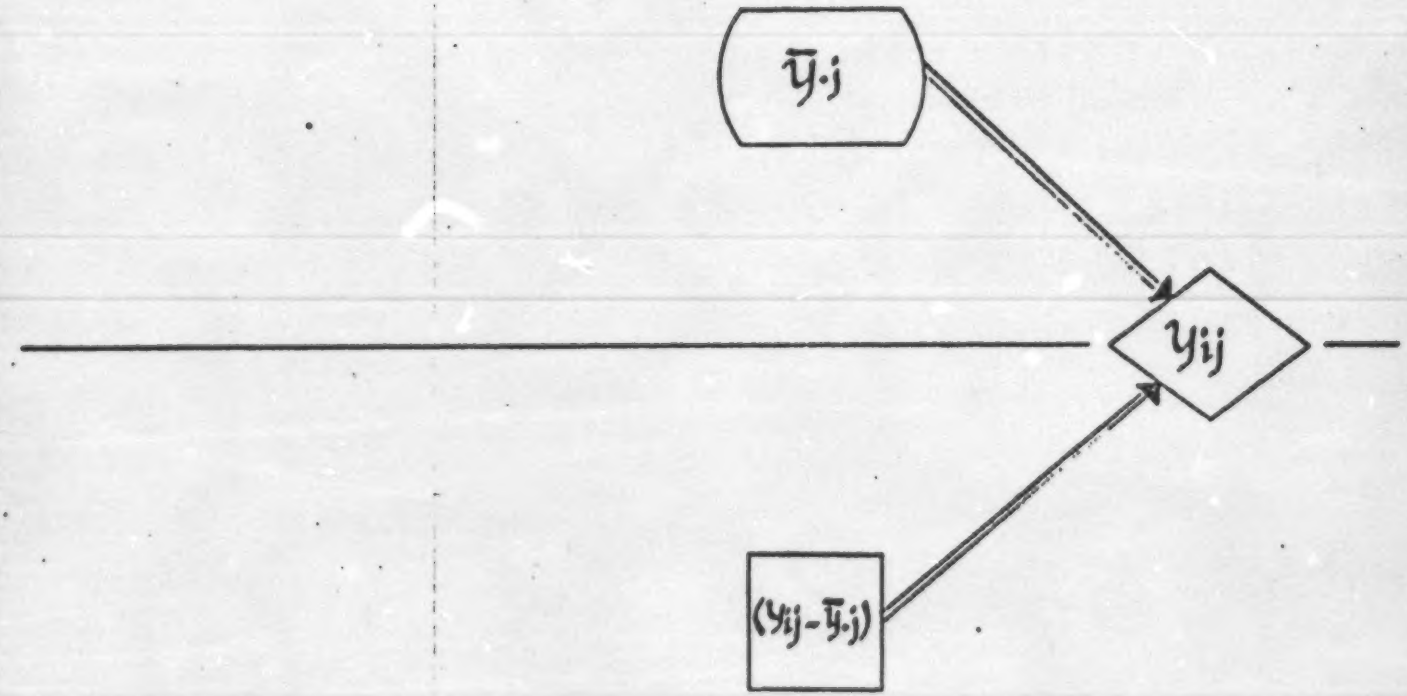
The second factor in each product here is a sum of deviations from a sample mean, and so is necessarily zero, while, for fixed values of j, the first factor in each summation is a constant. Since the second term is by construction one whose sample mean is zero, the orthogonality just proved also implies a zero covariance between the two terms. This is a very important property, because it implies that the two components of Y_{ij} may be treated separately and independently as long as only linear influences upon the outcome are being considered.

In similar fashion, any input variable (here, perhaps SES, or POP, or MOT) can also be algebraically decomposed into two components such that one of these components influences only $\bar{Y}_{.j}$ -- that is, it has zero covariance with $(Y_{ij} - \bar{Y}_{.j})$; and the other component influences only $(Y_{ij} - \bar{Y}_{.j})$ -- that is, it has zero covariance with $\bar{Y}_{.j}$. Also, these two components of X_{ij} exhaust all the influence of X_{ij} on Y_{ij}; they are themselves orthogonal to each other; and they have zero covariance with each other.

Making these decompositions enables us to organize the multi-level analysis in a neat and simplifying way. Figure 1, derived from Fennessey (1970), shows this decomposition of Y_{ij} in a way similar to that used in path analysis. Three differences between this diagram and those of path analysis are noteworthy: (1) in this and other similar diagrams, variables which are connected by a double line have their relationship fixed by the algebraic nature of the model. These double lines do not indicate causality, but instead a definitional relationship among the variables; this relation is an integral part of the model; (2) there is no error term in this diagram, since this decomposition equation is exact; (3) there is no curved arrow between the two components of Y_{ij}, since their covariance is known to be necessarily zero.

As just described, it is also possible to decompose the individual level explanatory variables as well as the individual outcome variable. Thus, in the example, SES, POP, and MOT each can be decomposed into a within-schools component and a between-schools component. Each of the within-schools components will be (because of the zero-sample-mean property) orthogonal to, and will also have zero covariance with, all of the between-schools components. Thus, this set of decompositions enables the investigator

FIGURE 2.1



to set up his work so that the multi-level analysis, at least in principle, reduces to two distinct analyses, one at the within-school level, and the other at the between-school level.

The principal analytic benefit of this approach is that it allows the investigator to treat separately the within-school mechanisms and the between-school mechanisms, as long as only linear relationships are considered, since the two sets of variables (deviations and means) are completely orthogonal.

While this is highly valuable, it is important to point out a conceptual issue that is often neglected in such discussions. The core of the problem arises in the fact that there are three distinct "outcomes" and "effects" of potential substantive interest, but they are linearly dependent. That is, the "observed," "between," and "within" outcomes, and the corresponding "effects," are constrained so that, given any two of them, the third can be algebraically computed. Thus, an empirical data analysis can shed light on at most two of the three distinct "effects."

Our thinking on this issue, and the present discussion, benefits greatly from a paper by Glenn Firebaugh, "Groups as Contexts and Frog Ponds: Some Neglected Considerations" (1977).

Suppose we have a variable X affecting some other variable Y, where X is a measure of ability, and Y is a measure of achievement. From the standpoint of substantive theory, it could well be that the absolute score on X (the "total" or "observed" component of X) affects the absolute ("total") score on Y. Second, the deviation $(X_{ij} - \overline{X}_{.j})$ (the "within" component) may affect the deviation $(Y_{ij} - \overline{Y}_{.j})$ (the "within" component) on Y. For example, if a person perceives himself as below the ability mean of his group, this

may reduce his ambition, or his perception of opportunity for rewards, and these within group processes may in turn lower his achievement score. Conversely, if a person is above his group's mean on X, this may tend to raise his self-esteem, and this in turn may raise his score on Y. This pattern would be an instance of what Firebaugh, and James Davis (1966) call a frog-pond effect. A third kind of process may also operate. In a group characterized by high average achievement, that average achievement may affect the pace and level of instruction presented by the teacher, and may also affect the standards of grading used; in addition, it may affect the informal peer norms as to minimum acceptable rates of gain. These processes will tend to affect the Y scores of all members of the group equally, and thus constitute a third category of effect (which Firebaugh refers to as a contextual effect).

The difficulty here is that there are potentially three categories of causal mechanisms operating: (1) processes involving the total X_{ij} score; (2) processes involving the group deviation score $(X_{ij} - \overline{X}_{.j})$; and (3) processes involving the group average $\overline{X}_{.j}$. But, these three terms are linearly dependent, and so any empirical analysis cannot determine the relative size of all three.

Two major comments can and should be made as a contribution to this issue, beyond simply recognizing it clearly and stating it plainly. First, the dilemma is, in principle at least, resolvable by a combination of (1) more detailed and explicit conceptualization and specification of the several processes involved, together with (2) the collection of more comprehensive data. That is, the investigator must specify in more detail the elements of each of the three kinds of processes being considered, and then must arrange to obtain evidence (usually data) as to the actual variables directly involved in each

process. Thus, by avoiding use of the deviation X or the average X as a proxy for some other conceptually more relevant variable, and by regarding the process as a multi-equation one, in which the direct causes of achievement levels are in turn indirectly and partly the consequence of the X variable, we can, through respecification, avoid the linear dependency, and can direct our attention more productively. This point, sensible though challenging to act upon, is made clearly, but perhaps not forcefully enough, by Firebaugh. It is, however, an exhortation to substantive purity and perfection.

The second comment is more in the nature of a justification for expediency. In the framework developed here, and consistent with the writings of Hauser, Fennessey, Alwin, and Cronbach, a choice has been made to deal with only two of the three elements, namely $(X_{ij} - \overline{X}_{.j})$ and $\overline{X}_{.j}$. This is equivalent to abandoning any attempt to measure the total, or absolute, impact of X on Y. Stated differently, by studying only the deviation and the mean, we must recognize the implication that any effects we observe, at either level, may be due in an unknowable part to that "total" relation of X to Y that we have suppressed. This is a limitation, but for our main purpose in carrying out a multi-level analysis, not a fatal defect. By definition, such "total" relationships are purely individual in nature; hence, it is entirely logical to posit that they are constant in strength across individuals and groups. Since the focus of our multi-level analysis is primarily sociological, the choice of analyzing only these two components amounts to possibly introducing a constant though unknown bias in our estimates of all the relationships. But, since it is a constant, such a bias does not, indeed cannot, affect the differences between coefficients

estimated in different settings. The bias makes the absolute size of the "within vs. total" coefficients problematic, but not differences of individual level coefficients across, or differences of group level coefficients across settings.

There are further ramifications of this notion that can and should be explored in a subsequent project, but in the present effort this particular clarification and agenda-setting is an important accomplishment, one which builds upon the recent work of Firebaugh.

We consider next one important limitation of this multi-level analysis framework, as originally developed. In order to apply fully the elegance of the formal model of analysis of covariance, on which the existing multi-level framework is based, it is necessary that there be no interactive effects between the individual level explanatory variables (i.e., the co-variates) and the school level explanatory variables (i.e., the "treatments"). Thus, as long as the framework is seen as that of analysis of covariance, one important type of multi-level process must be assumed out of existence. A more technical statement of the necessary condition is that the complete pattern of within-group slopes must be the same for all groups. As long as this assumption is valid, the individual deviation score on any explanatory variable can affect only the individual deviation score on the outcome. Similarly, the aggregated variable constructed from an explanatory variable can affect only the aggregated component of the outcome variable.

However, the assumption of equal within-group slopes is a strong one, and is often violated in actual data. The assumption <u>is</u> subject to empirical test, and such tests can be performed either on <u>all</u> the coefficients, or only on a subset of them, using the general approach of comparing the

predictive power of a specification which allows for distinct slopes with another specification in which several slopes are constrained to be the same. If the assumption of equal slopes is found untenable (as it often is) then some problems of interpretation may arise. If the basic specification is correct, then the most obvious conclusion is that the several schools are governed by distinct 'structures.' However, other possibilities likewise need to be considered. These too will be discussed.

The major advance incorporated in the initial versions of the linear decomposition paradigm for multi-level analysis (Hauser, 1968; Fennessey, 1970) was the clear distinction between compositional (or demographic) effects and "contextual" effects. Suppose (for the sake of the argument) that an individual's SES directly influences his ACH score, and that the deviation SES score also directly influences the deviation ACH score. Then, the coefficient we estimate for the within-school regression will be reflecting a combination of these two processes. To simplify, we will temporarily assume that the deviations have no influence, so that the estimated coefficient reflects only the overall individual level relation between SES and ACH. We further assume that the relation is linear in form. We specifically do not claim that the average SES in a particular school exerts any effect upon the average ACH in that school as a distinct process. If such an effect were present, it might be labeled a contextual effect.

Note that, even without considering a real and distinct contextual impact of the average SES in the school upon the average ACH, it would be clearly reasonable to expect the average ACH to differ from school to school if there are differences from school to school in the average value

of the SES variable. These differences would simply be a mathematical consequence of the individual-level equation process, together with the differing distributions of SES from one school to another. This notion, that observed differences between groups in the average score on an outcome variable may be due in part to corresponding differences between the groups in their average scores on an individual-level explanatory variable, is the basis of the use of analysis of covariance to adjust for what Cochran (1957) has called 'bias' in the estimate of the 'treatment' effect. What we are describing here is simply the mathematically necessary school-level consequence of differences between schools in their composition on SES. Such "demographic" effects (formerly labeled by Hauser and by Fennessey as "compositional") are often of analytic interest. However, once they have been clearly identified, the focus of attention shifts to an investigation of the reasons why the distribution of (say) SES differs from school to school. Interest in a multi-level analysis also concerns the truly school level processes as well; these correspond roughly to the 'treatment' effects in the analysis of covariance terminology. But, to estimate these effects in this technique, we must first adjust the school level outcome, $\overline{Y}_{.j}$, to remove the effects of differences in composition. Such an adjustment term has been referred to (Fennessey, 1970) as a composition adjustor.

As long as the model's assumptions apply validly to a particular set of data, this type of adjustment serves to compensate statistically for the "demographic effect" consequences of compositional differences between schools on the explanatory variables. There can be some fairly serious complications in actual use of this idea, since one can never be sure that the specified model is the truly correct one. Here we will merely indicate the general

type of problem that may arise when different influences are partly or entirely confounded. This discussion brings us close to the core of Robert Hauser's major objections to inferences about contextual effects.

Suppose for the moment that only a single explanatory variable, SES, is believed to affect ACH scores of individuals. Suppose also that the estimated regression coefficient is the same in all the schools. Let that coefficient be denoted as b_w. Then the adjusted school mean on achievement, denoted as $\bar{Y}^*_{.j}$, can be defined as

$$\bar{Y}^*_{.j} \equiv \bar{Y}_{.j} - b_w(\bar{S}_{.j} - \bar{S}_{..}). \qquad \text{eq. 2}$$

In the above equation, the relationship is an identity. The coefficient b_w was obtained separately from this equation, and thus the equation is simply a definition of $\bar{Y}^*_{.j}$. However, unlike the earlier decomposition equations, this one involves variables only at the between-school level, and the two terms on its right hand side are <u>not</u> necessarily orthogonal or uncorrelated. In fact, in general the two terms ($\bar{Y}_{.j}$ and $b_w(\bar{S}_{.j} - \bar{S}_{..})$) will be correlated.

The next step is to rearrange the terms of equation (2) as follows:

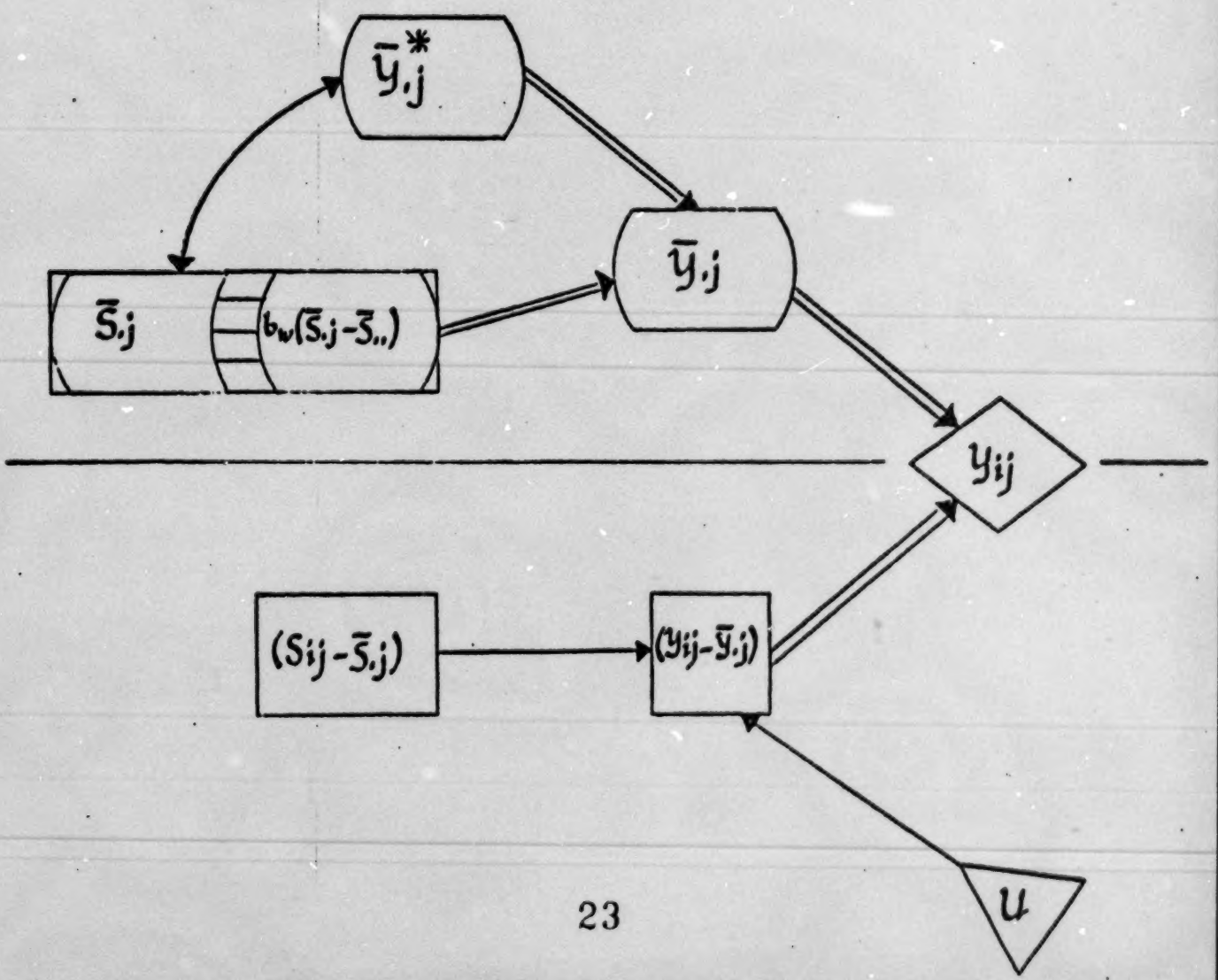
$$\bar{Y}_{.j} \equiv \bar{Y}^*_{.j} + b_w(\bar{S}_{.j} - \bar{S}_{..}). \qquad \text{eq. 3}$$

This gives an expression for $\bar{Y}_{.j}$ in terms of two components. Note that in equation (3) as in equation (2) the two terms on the right hand side are in general not orthogonal and are in general correlated. Such a correlation <u>can</u> be the result of what we will call a contextual effect (i.e., an actual impact of the group level of an explanatory variable, over and beyond the compositional consequences of that variable). Consequently, that correlation must be taken into account in the logic of the analysis, and also in the

diagrams. This decomposition of the school mean on ACH into a term due to composition and a term reflecting other influences is a distinctive feature of the framework or paradigm we are following. The expression $b_w(\overline{S}_{.j} - \overline{S}_{..})$ can be referred to as the covariance adjustor or as the composition adjustor. Equation (3) gives us two components of $\overline{Y}_{.j}$. Thus, we may explain variation in $\overline{Y}_{.j}$ by explaining the behavior of either component. In general, the two components may depend one on the other, or on any other school-level variables. This decomposition strategy thus avoids closing off possibilities. The term $\overline{Y}^*_{.j}$ becomes available to be explained, as well as theory and data will permit, by the available group level characteristics.

Having made these preliminary definitions, we present in Figure 2 an expanded (though still simplified) diagram showing the model under discussion at this stage. Note that the coefficient b_w has been obtained separately from the between-school model, and thus is treated in the between-school model simply as a known constant. The within-schools section of Figure 2 shows that two variables account for all the variation in $(Y_{ij} - \overline{Y}_{.j})$. The first of these is the individual deviation SES, and the second is an unmeasured disturbance variable, denoted U. In the between-schools section of the model, two variables again account for all the variation in $\overline{Y}_{.j}$. One of these is the composition adjustment term, $b_w(\overline{S}_{.j} - \overline{S}_{..})$, and the other is the implied school average, $\overline{Y}^*_{.j}$. The decomposition of $\overline{Y}_{.j}$ is complete, so there is no disturbance term for that portion of the model. The two components of $\overline{Y}_{.j}$ have a nonzero correlation, and this is indicated by the curved, double-headed arrow connecting them. Such a curved arrow is used conventionally in path analysis to indicate that a particular correlation is not assumed to be zero, and is accepted

FIGURE 2.2



as a given quantity. In later versions of this model, we shall introduce additional influences upon $\overline{SES}_{.j}$, including perhaps $\overline{Y}^{*}_{.j}$, thus respecifying the model so that the correlation between $\overline{Y}^{*}_{.j}$ and $b_w(\overline{S}_{.j} - \overline{S}_{..})$ will then be a candidate for explanation. In all of these decisions, a heavy reliance is placed on the relationships implied by the particular substantive theory being examined. In fact, one general conclusion resulting from our work is the extreme importance of precise theoretical specification in any attempt to examine data with multi-level models. This point will be elaborated further in later sections. We see a strong virtue in the careful decomposition of paths and patterns of possible dependence. The technique thus allows and encourages detailed theorizing. Responsibility for specifying actual process linkages, however, rests with the theorist. The technique is neutral or indifferent on this point.

The key idea is twofold. The particular adjustment elements used must ultimately be justified in terms of the investigator's best judgment about the concrete situation; there is no generally correct or incorrect procedure. Either overadjustment, or underadjustment, or even a complex combination of the two, are always possible. This point advances the discussion beyond that contained in the Wiley paper (1976). The only assurance one can have is that the <u>theory</u> being studied requires adjustment for certain terms and not for others. If, unknown to the investigator, that theory is incorrect, then the analysis will also be incorrect. But, general maxims as to how to adjust are of little value. What is needed, rather, is a consideration of how to detect instances of particular adjustment errors. Similarly, at the between-school level, there is no a priori rule that allows the analyst to decide whether or not to allow an arrow from any particular

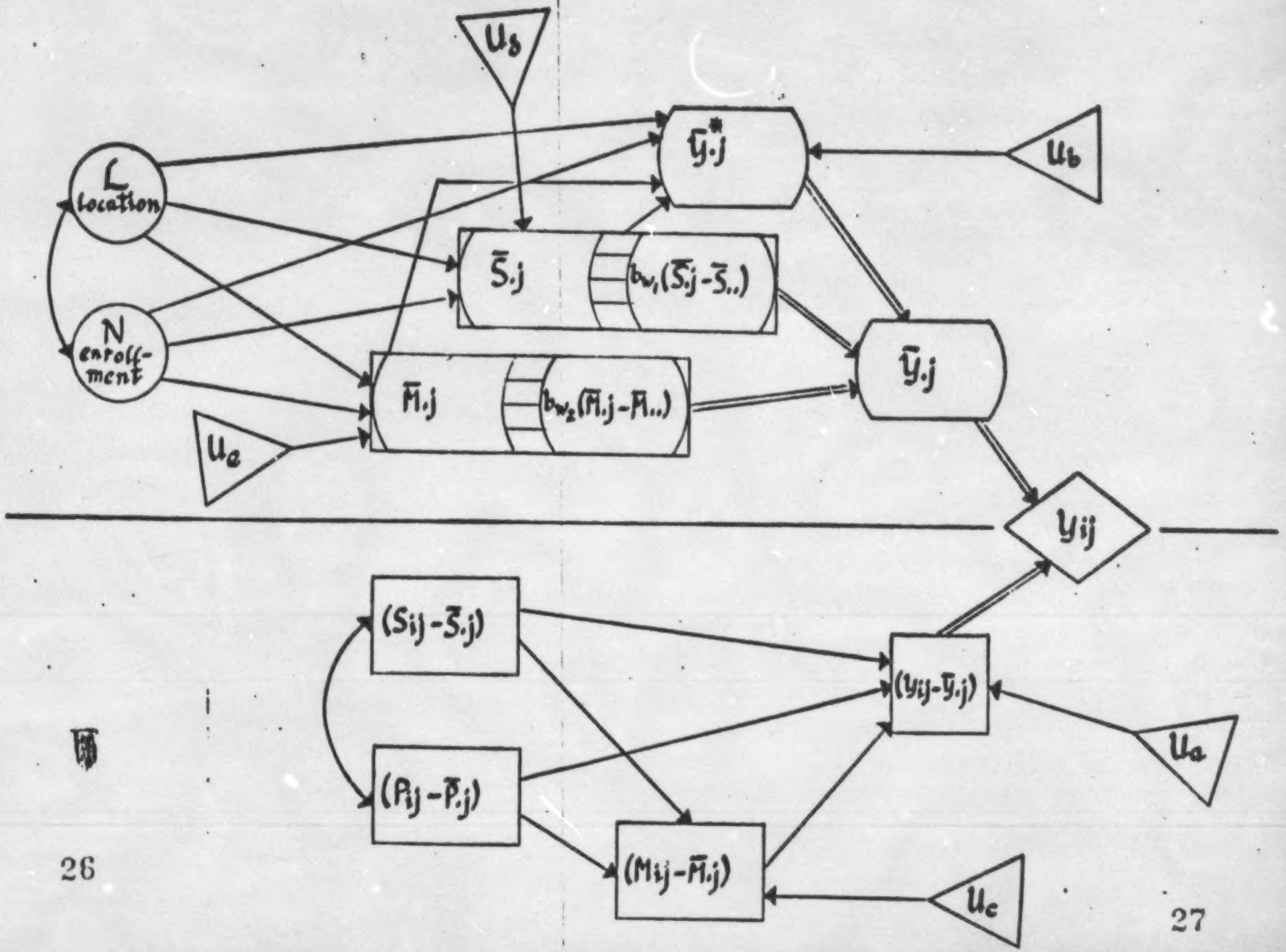
school-level property to $\bar{Y}^{*}_{.j}$; this decision is a judgmental one, based on the theoretical context and the research design.

Notice, however, that as a consequence of the orthogonality and thus the noncorrelation of the several variables used in the within-school model relative to those used in the between-school model, the two levels can be examined separately. Notice too that in discussion of the between-school level model, we have not become bogged down in questions as to whether the "treatment affects the covariate," which would have been recommended by the conventional usage of analysis of covariance. We have progressed beyond that, to a realization that the treatment and the covariate each are manifold entities, and the interrelations among their elements are problematic. Thus, approaching this multi-level task in terms of a multiple-equation linear model, whose elements include various decompositions and aggregations of the observed variables, and in which different equations involve variables of distinct types and levels, as we are doing, avoids one unnecessary constraint.

Figure 3 expands the model to include all the explanatory variables previously mentioned, and also illustrates a number of specific points. First, the reader should be clear that this is only one of many possible specifications that might have been examined. Choosing this particular set of relations was a deliberate action by the investigator, based on his/her judgment as to the most plausible relations.

At the individual level, the within-schools section of the figure indicates that there are three individual deviation variables affecting the deviation scores on ACH. Only two of these, namely SES and MOT, are also present as composition adjustors in the between-schools section of

FIGURE 2.3


U_a
$\bar{Y}^*_{.j}$
U_b
L location
$\bar{S}_{.j}$
$b_{w_1}(\bar{S}_{.j} - \bar{S}_{..})$
N enroll-ment
$\bar{M}_{.j}$
$b_{w_2}(\bar{M}_{.j} - \bar{M}_{..})$
$\bar{Y}_{.j}$
U_e
Y_{ij}
$(S_{ij} - \bar{S}_{.j})$
$(Y_{ij} - \bar{Y}_{.j})$
U_a
$(P_{ij} - \bar{P}_{.j})$
$(M_{ij} - \bar{M}_{.j})$
U_c

the diagram. The third variable, POP, appears only in the within-school model as a deviation variable. Its omission as a composition adjustor simply reflects the fact that the researcher decided (on whatever grounds) that there were no differences in average level of POP between schools. That particular omission, a questionable one in most circumstances, was made in this example to illustrate the fact that the specifications in the analysis are determined on substantive grounds. It will also simplify later exposition considerably, but this is only a tactical concern. Notice that this particular omission is also based on an assumption which can be rather easily tested, and thus a revised specification would be adopted if the data so indicate.

In the within-schools section of the model, there are two unmeasured disturbance variables, each indicated in the diagram by a triangle. These disturbance variables represent the combined effects of all the other factors which have not been explicitly included. As in any linear model analyses, certain assumptions are made about these U's. Note that there are in effect <u>two</u> structural equations being used in the within-cells section of the model. One equation has $(M_{ij} - \overline{M}_{.j})$ as the dependent variable; the other has $(Y_{ij} - \overline{Y}_{.j})$ as the dependent variable.

As formulations of sociological hypotheses, the relations in the diagram draw attention to several often obscure points.

For example, by including the global cell variables of school location and enrollment (shown in circles on the diagram) and by drawing arrows from each of them to the cell average on SES and on morale, we have explicitly specified that these "treatment" variables do affect these "covariates." This specification is in no way required by the abstract model, but is a

specific substantive assumption freely made, and one which can be examined in the course of the research. It is a benefit of the structural equations conception instead of the ANCOVA one. We explain, partly, the compositional process this way.

According to the diagram, $\overline{Y}^{*}_{ij}$ is partly determined by a causal influence from the cell averages, $\overline{S}_{.j}$ and $\overline{M}_{.j}$. The inclusion of these arrows is equivalent to the postulation of a distinct school level effect, one which may be labeled a contextual effect. This effect, which operates strictly at the school level, logically flows from the school average, and is distinct from the composition adjustment and its demographic effect. It should be made clear that these arrows are in no way logically required, but rather are expressions of the substantive hypotheses in this particular piece of research. Symbolically, what is being maintained is that the $\overline{S}_{.j}$ influences $\overline{Y}^{*}_{.j}$, which in turn is a component of $\overline{Y}_{.j}$, which in turn is a component of Y_{ij}. Of course, the latter relations are not strictly causal, and so the diagram notation was broadened (double lines) to cover them. In this formulation, the meaning of what is often a hazy concept, namely that of contextual effects, is precisely located.

By showing no direct link between the composition variables for SES and the one for morale, and also no correlation between U_d and U_e, we have equivalently postulated that the entire between-schools covariance between the two composition variables, S and M, is accounted for by their mutual dependence on L and N, the variables of location and enrollment, respectively. Note that in this model, one is able to postulate an explanation for the covariance between the aggregative cell variables derived from SES and morale, but at the same time leave their individual deviation form unexplained.

One other feature of the model which is brought out by this paradigm is that the global variables may be either continuous or discrete. It also should be emphasized that other aggregative statistics in addition to school means, for example the SES diversity of the students in a school (defined as the standard deviation of the students on the SES scale) could be used in the model. Also, a variable which is inherently aggregative can be classified as global if the corresponding individual level variable is not included in the model. This however is a very tricky situation (cf. Hauser, 1970, 1974). Thus, it is the use in the model which determines the way a variable is classified, and not the way the variable is measured.

In working with this paradigm, it is necessary first to test whether the entire pattern of slopes on all the within-cells variables is the same in each cell, that is, a test for interaction. Intuitively, this is carried out using the general logic of significance testing for an additional set of variables in a linear model using an F-test. It may be necessary to carry out separate regressions within each school, adding the sum of squares obtained, and comparing that (after division by the appropriate degrees of freedom) with the sum of sqaures obtained using only a single equation (again after dividing by the appropriate degrees of freedom).

One limitation of the model is that it cannot be applied unless the dependent variable is measured on at least an interval scale. This requirement, like that concerning equality of slopes across cells, is one which holds quite generally, and is not specific to the linear decomposition multi-level analysis framework. Also, it is sometimes possible by the use of transformations or by a redefinition of the variables of interest, to create an interval-scale dependent variable when an original dependent variable is not measured on an interval scale (e.g., logit or probit analysis).

Another set of problems which may arise in this model is one which does not arise in single equation linear models, but which is implicitly a part of any method which handles simultaneous equation models, namely the matter of identifiability of partifular coefficients and equations. Sometimes, such equations simply cannot be estimated from the data at hand (underidentified). In other situations, there is an excess of information, with the result that more than one solution is possible (over-identification).

But, of more interest than the limitations--which are in no way peculiar to this application of the simultaneous linear model--is the interpretation of various patterns in the data, and the development of a clear understanding of the kinds of effects which can occur.

The most obvious kind of multi-level effect is what can be called a pure demographic effect. The meaning we shall give to this and other terms is not necessarily the same as previous usage, so the reader should be alert to this fact. However, the point is not what label is applied, but rather that the phenomenon be understood. The essence of a demographic effect, in this terminology, is that it is not a result of the social structure (i.e., the school) being considered, or the ecology. That is, it can be achieved simply by a priori partitioning of a large sample into a number of smaller samples, each of which has a different average value on some independent variables. This is the sort of effect which is adjusted out by the composition adjustor.

The next sort of effect to be considered is what for the moment we call a contextual effect. Its characteristic is that it changes the level of some variable for all persons in the group by a constant amount, but does not affect the relationships between any variables within groups. This corresponds approximately to the "treatment effect" in the analysis of covariance,

and is the type of effect which can be well studied using the analytical model described in the preceding pages. It does indicate a real effect of the social structure, one which could not have been produced simply by arbitrary combining of individual cases into an aggregate.

The third type of effect to be considered is one which seems best labeled as a structural effect, in keeping with the terminology of econometrics. In it, the coefficients of the substantively meaningful equations are described as the structure. This effect of a macro-social structure, in other words, operates by changing the form of micro-structural relationships among variables. It corresponds to the "unequal slopes" situation in traditional analysis of covariance, and creates a situation in which there are no further codified procedures available in traditional analysis of covariance.

Two other points need to be reiterated. First, the characteristics of different groups, which make them have their particular effect, include not only averages or strictly global variables, but also include the degree of dispersion on some characteristics within the group. In fact, any aspect of the distribution of characteristics in a social group, including covariances among different characteristics, can be conceptualized as responsible for one or another demographic, contextual, or structural effect. The second point is that it is also possible to consider the size of R^2 or σ^2_μ within each school as an outcome of macro-influences. That is, this too measures a meaningful aspect of the group, and if it varies from one group to another, then we have found still another type of multi-level effect. This last notion was not pursued in the present project.

One difficulty in making confident inferences about the kind of effects we have called macro-level effects arises because omission of an important variable in the individual model (the within-schools model) will mean that the differences in distribution of that variable from school to school are not considered as a possible cause of difference in $\bar{Y}^{*}_{.j}$. This, of course, is not an unusual limitation and corresponds quite directly to the familiar caution that any linear model is fully interpretable only if it is the correctly specified model for the data. The special feature is that there are, in a multi-level model, additional confounding variables beyond the strictly global ones, or the true aggregative effects. These others are the additional individual level variables which would also have an indirect, compositional effect. This issue has been clearly defined by Hauser (1970).

Section 3. Advances in this Project - The Linear Decomposition Framework

Our work to synthesize and build upon these ideas has been a sometimes slow process, with numerous intertwined issues, and the frequent feeling that one is threading one's way through a maze, never quite knowing what lies beyond the next bend in the road. In addition to the relatively specific objectives (alternative adjustment strategies; using the vector of coefficients as an outcome multivariate observation within a generalized least squares analysis), we have devoted much time to seeking out an adequate general framework. Several directions have been pursued, including the recent work of Leigh Burstein (1975, 1976), that of Lee J. Cronbach and his associates, the general idea of variance components modeling (as seen in the writings of Ward Keesling (1974)). As we began drafting the final report, late in 1978, we undertook one additional reappraisal of the issues before acknowledging that no better framework seemed available. That examination, in which we sought a somewhat radical generalization of the potential in the existing linear structural equation framework for the multi-level analysis, led to the discovery of a new approach. We adopted a change in heuristic, and it made possible the discovery we sought. Rather than examining the available statistical paradigms further to see whether and how they might be adapted to the requirements of the multi-level analysis situation, we instead sought a framework which could cleanly and explicitly represent the various kinds of multi-level processes and effects our work has led us to recognize. We sought also to represent the interconnections among those effects. The remainder of this section describes that new approach, which we have called the linear decomposition framework.

A LINEAR DECOMPOSITION FRAMEWORK FOR MULTI-LEVEL ANALYSIS

3.1 Introduction

Social science researchers often confront a situation in which several distinct processes are postulated as operating, each at a separate (usually nested) level of aggregation. For example, in studying school achievement processes, the substantive model being used may posit that certain influences (e.g., mental ability, motivation, home socioeconomic status) operate on each individual student; other influences (e.g., teacher competence, allocation of classroom time) may be posited to operate identically on all students in a single classroom, but differ from one classroom to another. Still other influences (e.g., principal's leadership, community support) may affect all the students in a single school identically, but differ across schools. If this sort of model were applied in an experimental situation, it would call for some form of nested analysis of variance. However, when the study is an observational one, and when the theoretical context requires the possibility of interdependence and reciprocal causation among different variables, the choice of appropriate analysis techniques becomes highly problematic.

There has been a fair amount of work on questions of this sort by sociologists, but the level of available techniques is still rather restricted. A recent paper by Alwin (1976) identifies two broad approaches to this sort of question (labeled as the contextual analysis approach

and the analysis of covariance approach) and shows that they are fundamentally equivalent. To date, however, each of these approaches has been developed only for two levels of aggregation, and only for single-equation models. Furthermore, neither handles in a satisfactory way the possibility of differing slope patterns from school to school. In the present paper, we outline a more general approach, one suited to handle multiple levels of aggregation, multiple equation models, and varying slopes.

The basic rationale of this approach is to develop a substantively meaningful linear decomposition of the outcome variable of main interest, and corresponding decompositions of the input variables believed to be relevant. The several components of the outcome variable, and those of the input variables, then are treated as distinct variables in a general structural equation model. The analysis of such a model, including the representation of reciprocal influences and known dependencies, then can be carried out rather directly using Joreskog's LISREL computer program (Joreskog and Sorbom, 1978).

The general approach of decomposing a variable into convenient and/or meaningful components is a widely used one, but its generality is seldom fully appreciated. It includes work along the lines illustrated in Duncan (1966) and in Stinchcombe (1968), and also techniques such as factor analysis. Any such decomposition approach is to be judged by the degree to which it enhances both the substantive clarity of the research questions, and the actual convenience and tractability of the quantitative analysis. For example, the early version of the linear decomposition framework just discussed has created two meaningful components of the outcome variable and of the input variables, and has exploited the fact

of orthogonality among several of these components to simplify considerably the multi-level analysis.

To present the ideas underlying this new framework, we will make use of a simple, illustrative example. The example does not indicate the full potential of the technique, but does serve to convey its main ideas.

3.2 An Illustration of the Linear Decomposition Framework

For the main illustration, we use a single equation with a single regressor variable. This is not intended to be a realistic example, but it is simple enough to allow the logic of the decomposition to appear. Suppose a researcher is seeking to estimate the influence of socioeconomic status (SES) on the academic achievement (ACH) of a sample of N students, with those students distributed across a number of schools (let JMAX denote the number of schools). Thus, this example involves only two levels--individual students and schools. In discussing the model, we shall use a notation scheme adapted to computer programming. While less terse and general than the usual mathematical symbols, it is more concrete, and also easier to prepare typographically.

To begin, we note that the observed score of student I in school J can be decomposed into several components:

(1) $$\text{ACHOBS}(I,J) \equiv \text{ACHGAVG} + \text{ACHDEV}(J) + \text{ACHDEV}(I,J)$$

The first term is an overall grand average; the second is the deviation of the average in school J from that grand average; and the third term is the deviation of the score of the I^{th} student in the J^{th} school from the average for the J^{th} school. The relationship stated in equation 1 is an identity. That is, it is non-causal, exact, and definitionally true; it

involves no probability distribution. Also, each of the terms on the right hand side is orthogonal to, and uncorrelated with, the other two terms.

The input or regressor variable, SES, may be decomposed into a parallel set of components:

(2) $\quad SESOBS(I,J) \equiv SESGAVG + SESDEV(J) + SESDEV(I,J)$

For this equation, as for equation (1), the terms on the right hand side are pairwise orthogonal and uncorrelated. Moreover, each such term of equation (2) is correlated only with the corresponding term of equation (1). This localizing of the correlations enables us to organize the multilevel analysis in a highly elegant and convenient way. In particular, the ACHDEV(J) component is orthogonal to, and uncorrelated with, the SESDEV(I,J) component, and the ACHDEV(I,J) component is orthogonal to, and uncorrelated with, the SESDEV(J) component.

Taking advantage of these decompositions and the accompanying orthogonality, one first carries out a routine linear model analysis (using ordinary least squares) of the ACHDEV(I,J) scores regressed on the SESDEV(I,J) scores. This calculation raises the possibility that the several different schools will be characterized by distinct slopes. In practice, this frequently occurs, but the usual analysis of covariance logic provides no adequate way to deal with it. We shall later show how the framework improves matters in that respect, but for the moment, we temporarily assume that the several slopes are identical. That slope will be denoted BWAVG--standing for the average within-school slope pooling over schools.

In the earlier work with the analysis of covariance approach to multi-level analysis, one major advantage was the clear distinction be-

tween two kinds of effects: (1) compositional or demographic effects; and (2) 'contextual' effects. The demographic effects (called 'compositional' by Hauser [1968] and by Fennessey [1970]) are simply the mathematical consequence of the individual-level influence of SES on ACH, together with the fact that average SES differs from one school to another. In analysis of covariance terminology, this would be referred to as the 'bias' in the estimated treatment effect (cf. Cochran, 1957). Once such demographic effects have been recognized, and numerical estimates of their magnitude computed, the scientific investigatory interest shifts to a concern with the mechanism which creates differing averages on SES from school to school.

The other type of effect, 'contextual,' is more truly a school-level outcome. It corresponds to the 'adjusted treatment effect' in the usual analysis of covariance. To estimate these treatment effects, we must first adjust out the demographic effect. Note that the quantity remaining after the demographic effect has been removed is simply a residual; some or all of it may be the result of a contextual effect, but this is an empirical question. To remind ourselves of this, we use the symbol REM(J) to denote that quantity in school J. Thus, we have:

(3) $\quad REM(J) \equiv ACHDEV(J) - BWAVG*SESDEV(J)$

This is also an identity. The coefficient BWAVG is treated as a known constant (its value was obtained from the previous calculations on the within-school data). We can now transpose terms from equation (3) to yield a new, further decomposition of ACHDEV(J), namely:

(4) $\quad ACHDEV(J) \equiv REM(J) + BWAVG*SESDEV(J).$

In equation (4), the terms REM(J) and BWAVG*SESDEV(J) are in general correlated; there is no general reason to presume them to be orthogonal. Equation (4) thus partitions ACHDEV(J) into a term due to demographic effects of compositional differences, and another term summarizing all the remaining influences at the school level, namely REM(J).

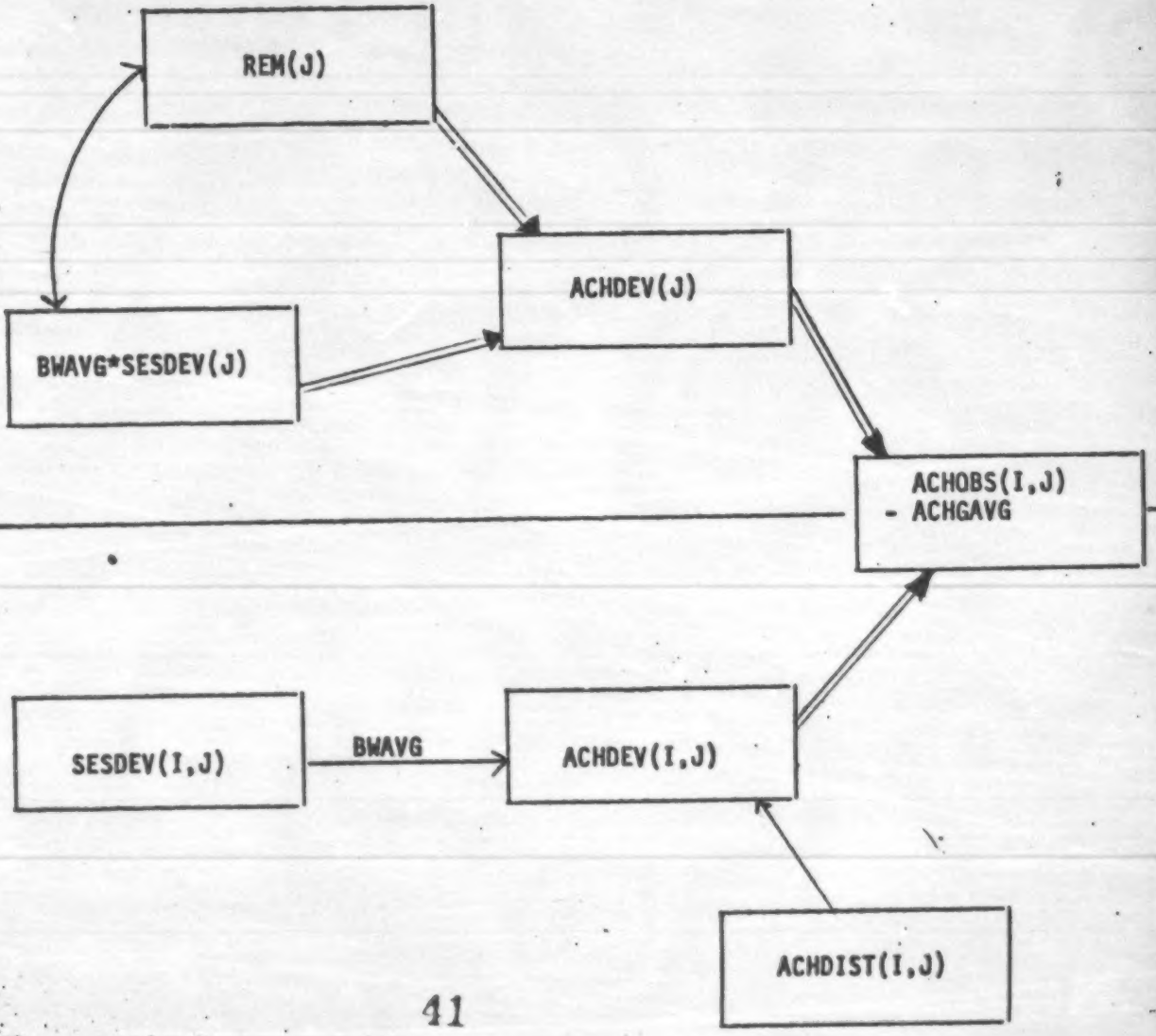
Notice that the nonzero correlation between REM(J) and BWAVG*SESDEV(J) becomes itself a feature to be explained. Thus, the between-school model now allows us to represent various influences upon either REM(J) or SESDEV(J), perhaps including reciprocal effects of one upon the other. This formulation is thus neutral and flexible. It allows--and indeed forces--the investigator to choose explicitly what patterns of influence will be postulated at the school level. This corresponds appropriately to the demands of real scientific work. In actual research, there can be no general rule to determine what influences might affect REM(J) and what might affect BWAVG*SESDEV(J). Thus, no general adjustment formula can be universally applicable. Only in relation to a specified theory is the possibility of overadjustment or underadjustment meaningful. This point is not appreciated in Wiley's (1976) discussion of related topics.

Before going on to posit further influences upon REM(J) and BWAVG*SEDEV(J), we present in Figure 1 the simplified model as it has been thus far developed. The figure shows that the two components of ACHDEV(J) are correlated and thus may conceivably influence each other.

FIGURE 1 about here

The diagram also makes clear that the within-school portion of the model

FIGURE 3.1


REM(J)
ACHDEV(J)
BWAVG*SESDEV(J)
ACHOBS(I,J)
- ACHGAVG
SESDEV(I,J)
BWAVG
ACHDEV(I,J)
ACHDIST(I,J)

is separate from the between school portion. The within-school portion includes a disturbance variable, and thus accounts fully for the behavior of ACHDEV(I,J).

So far, we have developed a fairly simple linear decomposition analysis for a multi-level investigation. Recall that we have temporarily assumed that the slope of ACH on SES is the same in each school. If this assumption were not true, there would be a third type of mechanism operating to determine the value of ACHDEV(J). This third type of effect--the consequence of differing slopes from school to school--will be called a structural effect. The conceptualization of this sort of effect was developed in the early work by Hauser and by Fennessey, but was not merged into the formal analysis model. Our present work has carried the analysis of this notion further, and will be the subject of the next several paragraphs.

To introduce the possibility of school-specific slopes, we can modify the decomposition contained in equation (4). Instead of a common slope BWAVG, we will use BW(J) to denote the slope in the J^{th} school. Thus, we have:

(5) $\quad ACHDEV(J) \equiv REM(J) + BW(J)*SESDEV(J)$

The difficulty with this equation is that the second term on the right hand side lumps together the demographic effect consequences of differences in school SES, as reflected in SESDEV(J), with the structural effect consequences of slope differences between schools, as reflected in BW(J). Thus, a further decomposition is needed.

To pursue this, we introduce a decomposition of the school-specific slope as follows: $BW(J) \equiv BWAVG + BWDEV(J)$. Thus, we reintroduce a

single overall summarizing slope, and at the same time introduce a counterpart quantity that is the deviation of the slope in the J^{th} school from that overall slope. BWAVG is the overall slope, and BDEV(J) is the deviation slope in school J; the monograph by Cronbach et al. (1976) uses a similar notion. With this definition of the slope, equation (5) can be rewritten as follows:

(6) $\quad ACHDEV(J) \equiv REM(J) + BWAVG*SESDEV(J) + BDEV(J)*SESDEV(J).$

With this equation, the first component on the right hand side appropriately indexes the remainder quantity; the second component on the right hand side appropriately indexes the demographic effect. However, the third term on the right hand side defines a component that still includes not only the structural effects but also the consequences of demographic patterns, and of the alignment between slopes and means. Therefore, still further decomposition is in order.

We achieve this further decomposition by making use of an algebraic trick. We subtract and add the quantity BDEV(J)*SESGAVG to the right hand side of equation (6), and then rearrange the terms. Thus, after this rearrangement, we have:

(7) ACHDEV(J) ≡	REM(J)	remainder component
	+ BWAVG*SESDEV(J)	demographic component
	+ BDEV(J)*[SESDEV(J) - SESGAVG]	alignment component
	+ BDEV(J)*SESGAVG	structural component

What has been accomplished by equation (7) is the substitution of four components for the focal outcome observation at the school level, ACHDEV(J). These components have some interesting properties. First, they involve

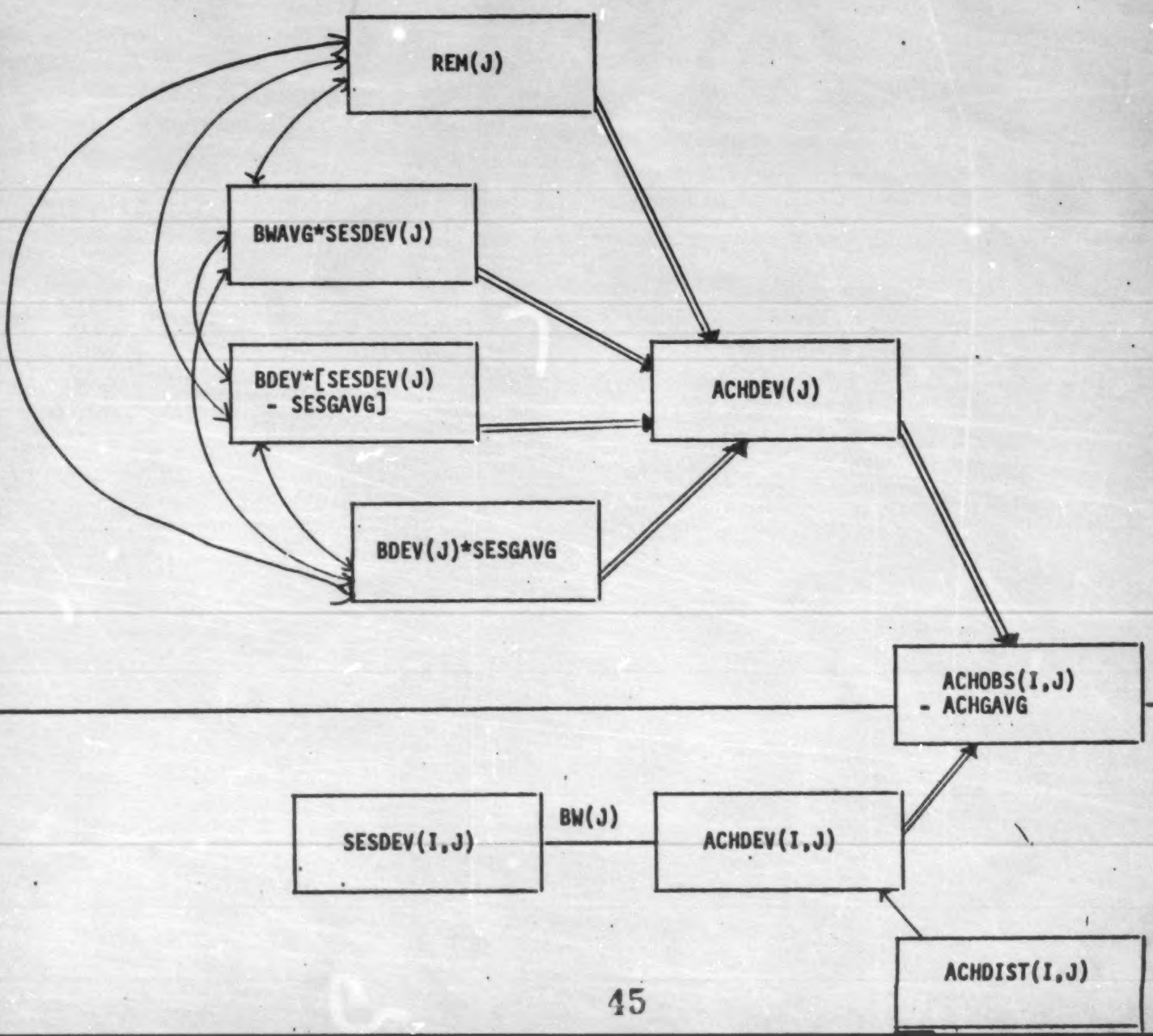
some definitional conventions. That is, the impact of slope differences is defined as being measured at the overall mean on the regressor variable (here, SES). Similarly, the impact of school to school differences in average SES level is defined as being measured at the mean slope (BWAVG). Second, notice that there is one component in which only the slopes vary, a second component in which only the SES levels vary, a third component in which both the slopes and the levels vary, and a fourth component in which neither the slopes nor the levels appear. A third noteworthy property is that in general these four components of ACHDEV(J) are intercorrelated. There is no priori reason to posit that they will be orthogonal or uncorrelated. Thus, as with the simpler decomposition presented earlier in Figure 1, the intercorrelation among the components become a feature to be explained if possible. And, once again, the formulation is neutral and flexible. It can accommodate a wide variety of influence patterns.

Figure 2 presents diagrammatically this more detailed and flexible decomposition.

FIGURE 2 about here

The notion of an 'alignment' term is a novel one. At first glance, this term--which is in the form of a product--may be taken to reflect a kind of interaction. However, that is not the case. The more appropriate analogy is that of decomposing the variation in a random variable made up as the sum of two other random variables, and having a covariation term. The alignment term in our decomposition indicates not an interactive effect, but the degree of co-occurrence of two distinct sources of variation. To

FIGURE 3.2


REM(J)
BWAVG*SESDEV(J)
BDEV*[SESDEV(J)
- SESGAVG]
ACHDEV(J)
BDEV(J)*SESGAVG
ACHOBS(I,J)
- ACHGAVG
SESDEV(I,J)
BW(J)
ACHDEV(I,J)
ACHDIST(I,J)

the degree that steep slopes occur more often in schools with, say, high values on SES, those schools will have a higher outcome score than otherwise predicted. This would be an example of positive alignment of slope and SES level. This separation of the alignment component from the remainder component allows a more detailed discussion of the processes.

Notice that if SESGAVG = 0, then the structural component becomes zero, and the alignment component alone captures all the effects of the varying slopes from school to school. This is a somewhat undesirable feature of the decomposition, but is not avoidable. The impact of slope differences must be measured at some value of the input variable; the overall mean is the most defensible choice. However, an implication of this is that this decomposition logic is <u>not</u> independent of the origin chosen for the regressor variable.

Similarly, if BWAVG = 0, even though the slope coefficients do fluctuate, then the demographic component becomes zero, and only the alignment component captures the effect of the varying school averages on SES.

The implication of this seems to be that the alignment component absorbs some of the demographic effect and some of the structural effect, and that it may in particular applications capture all of one or the other, depending upon the origin points used for the ACH and SES variable. Notice that if the ACH variable and the SES variable are "rotated" the slope BWAVG can be made zero. In other words, the allocation of variation to the structural component, the demographic component, and the alignment component is somewhat a matter of choice of frame of reference. For example, any constant and not just SESGAVG could be used as the reference

point for defining the structural effect and the alignment effect. Changing that constant will change the proportion of variation in each component, but the sum will remain fixed. One rationale for the choice of SESGAVG as the constant is that, in a certain sense, this minimizes the variation due to the alignment component. In other words, if we knew nothing about the slopes, but wished to minimize the sum of crossproducts that is the alignment term, our most reasonable choice of constant would be the grand mean, since this choice makes the sum of the second factor zero. Thus, under an assumption of constant slopes (i.e., all BW(J) are the same) only SESGAVG as the constant would yield the intuitively appropriate result, namely zero, for the alignment term. These arguments are not decisive, but they are persuasive; together with the recognition that the decomposition is dependent upon the location of the X and Y variables, they help our understanding.

Now that we have created this set of components, we may use the LISREL program (Joreskog and Sorbom, 1978) to analyze them. That program is designed to accomplish the maximum likelihood estimation of multi-equation linear models, including the possibilities of reciprocal causation, and also imperfect measurement and factorial structure between some indicators and the theoretically relevant constructs. For our discussion today, we shall ignore any measurement issues, and proceed as if all the variables under discussion are measured satisfactorily.

The LISREL program distinguishes between exogenous variables (labeled as KSIs) and endogenous variables (labeled as ETAs). It also allows representation of a set of structural disturbance variables (labeled as ZETAs). It postulates a covariance matrix among the exogenous KSIs, which is

denoted PHI. Similarly it postulates a covariance matrix among the disturbance ZETAs, which is denoted PSI. Finally, it postulates two separate matrices of coefficients. The first matrix, denoted GAMMA, contains the structural coefficients relating the endogenous ETAs to the exogenous KSIs. The second matrix, denoted BETA, contains the coefficients relating the endogenous ETA variable to one another.

In our use of the program, the four components of ACHDEV(J) are treated by LISREL as four endogenous variables (ETAs). A fifth endogenous variable may be included, which is ACHDEV(J). If that is done, then the row of the BETA corresponding to ACHDEV(J) is filled with known coefficients indicating the form of the combination is known, and the diagonal element of the PSI matrix corresponding to ACHDEV is set to zero, indicating that the relationship is an exact, known one.

To illustrate more concretely the kinds of flexibility that exist in the linear decomposition framework together with the LISREL analysis package, suppose we introduce a new variable, the principal's leadership ability, denoted LEAD(J). This will be a strictly exogenous variable in the linear structural model. In LISREL terminology, it will be KSI(1). Following the sequence indicated in Figure 2, we will use ETA(1) to refer to REM(J); ETA(2) to refer to BWAVG*SESDEV(J); ETA(3) to refer to BDEV(J)*[SESDEV(J) - SESGAVG]; and ETA(4) to refer to BDEV(J)*SESGAVG. Thus, for the between-schools portion of the multi-level analysis, we have a four-equation linear model. The real advantage of the LISREL package at this stage is that it presents the researcher with the BETA matrix and the GAMMA matrix, and requires that each element of those matrices be de-

fined either as fixed and known, or as unknown and to be estimated. The BETA matrix thus contains all the possible lines of direct influence between all possible pairs of components; the GAMMA matrix contains all the possible lines of direct influence from KSI(1) to each of the ETAs.

The example includes only one exogenous variable, and four endogenous variables; this relatively large number of endogenous variables in proportion to the number of exogenous variables makes questions of identifiability of serious concern. However, the main points of the illustration

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FIGURE 3 about here

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can still be made. Figure 3 contains an example of how the BETA and GAMMA matrices might be specified. Taken together, row 1 of the BETA matrix and row 1 of the GAMMA matrix indicate that the REM(J) component depends only on LEAD(J), and not on any of the other components of ACHDEV(J). In short, the model postulates a direct effect of principal leadership upon achievement. This is <u>not</u> what is usually meant by a contextual effect. However, if the covariation between REM(J) and ACHDEV(J) is larger than can be accounted for by the path through LEAD(J), then a reasonable inference would be that <u>part</u> of the REM(J) component does represent an actual contextual effect. This statement, of course, is contingent upon all the premises of the posited model, and should not be misunderstood as an absolute claim of having demonstrated that a contextual effect is the only possible explanation. Row 2 of the BETA and GAMMA matrices indicates that the demographic component of ACHDEV(J) also depends only upon the principal's

FIGURE 3.3

LISREL BETA MATRIX

			REM(J)	BWAVG*SESDEV(J)	BDEV(J)*[SESDEV(J) - SESGAVG]	BDEV(J)*SESGAVG
			(1)	(2)	(3)	(4)
remainder	REM(J)	(1)	1.000	0	0	0
demographic	BWAVG*SESDEV(J)	(2)	0	1.000	0	0
alignment	BDEV(J)*[SESDEV(J) - SESGAVG]	(3)	0	0	1.000	0
structural	BDEV(J)*SESGAVG	(4)	0	β 42?	0	1.000

LISREL GAMMA MATRIX

			LEAD(J)
			(1)
remainder	REM(J)	(1)	Γ_{11} ?
demographic	BWAVG*SESDEV(J)	(2)	Γ_{21} ?
alignment	BDEV(J)*[SESDEV(J) - SESGAVG]	(3)	Γ_{31} ?
structural	BDEV(J)*SESGAVG	(4)	0

leadership. In simple terms, wealthy families seem to move to schools whose principals are skilled leaders. Row 3 posits that, in addition to this, the leadership qualities of the principal also tend to produce an alignment between the levels and the slopes, and that this is the only influence upon the alignment component. Finally, row 4 of the BETA and GAMMA matrices indicates that the structural component is affected by the demographic component. In other words, the presence of high SES students tends (say) to make the slope of achievement on SES steeper. The fixed values of zero in other places in the BETA and GAMMA matrices indicate the scientist's specification that these particular possible influences are non-existent.

This example, simple and perhaps unrealistic, nevertheless does show that the Linear Decomposition Framework is practical and useful. In more complex and realistic models, it would still be usable. Although it is just recently developed, it appears to offer serious promise for clarifying what has been one of the murkiest areas of social science analysis.

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